15. Triangles and their Properties

- A triangle can be classified on the basis of the measures of its angles and sides.

- Classification of triangles on the basis of the measures of its angles:

| Name	Nature of the angle
Acute-angled triangle	Each angle is acute
obtuse-angled triangle	One angle is obtuse
Right-angled triangle	One angle is a right angle

- Classification of triangles on the basis of the lengths of its sides:

Name	Nature of the angle
Scalene triangle	All three sides are of unequal length
Isosceles triangle	Any two sides are of equal length
Equilateral triangle	All sides are of equal length

- **Angle sum property of triangles:**

  The sum of all the three interior angles of a triangle is 180°.

  $\therefore \angle A + \angle B + \angle C = 180°$
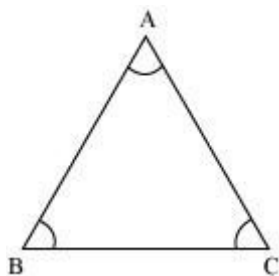
  Example:

  If the measures of the angles of a triangle are in the ratio 2: 4: 6, then find all the angles of the triangle.

  Solution:

  Ratio of the measures of angles = 2: 4: 6

  Therefore, let the angles of the triangle measure $2x$, $4x$, and $6x$.

Now, $2x + 4x + 6x = 180°$ {By angle sum property of triangles}

$\Rightarrow 12x = 180°$

$\Rightarrow x = 15°$

Thus, the angles of the triangle are

$2x = 2 \times 15° = 30°,$

$4x = 3 \times 15° = 60°$
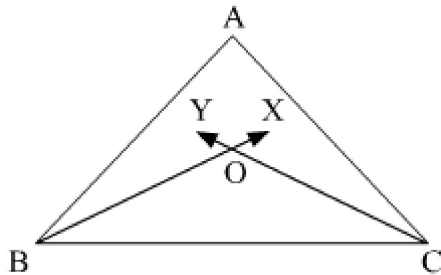
$6x = 6 \times 15° = 90°.$

The measure of one of angle is $90°$.

- **Facts deduced from angle sum property of triangles:**

    There can be no triangle with two right angles or two obtuse angles.
    There can be no triangle with all angles less than or greater than $60°$.

- **Relation between the vertex angle and the angles made by the bisectors of the remaining angles:**

    In $\triangle ABC$, BX and CY are bisectors of $\angle B$ and $\angle C$ respectively. Also, O is the point of intersection of BX and CY.

    Therefore, $\angle BOC = 90° + \frac{1}{2}\angle A$.